Stealing Watermarks of Large Language Models via Mixed Integer Programming

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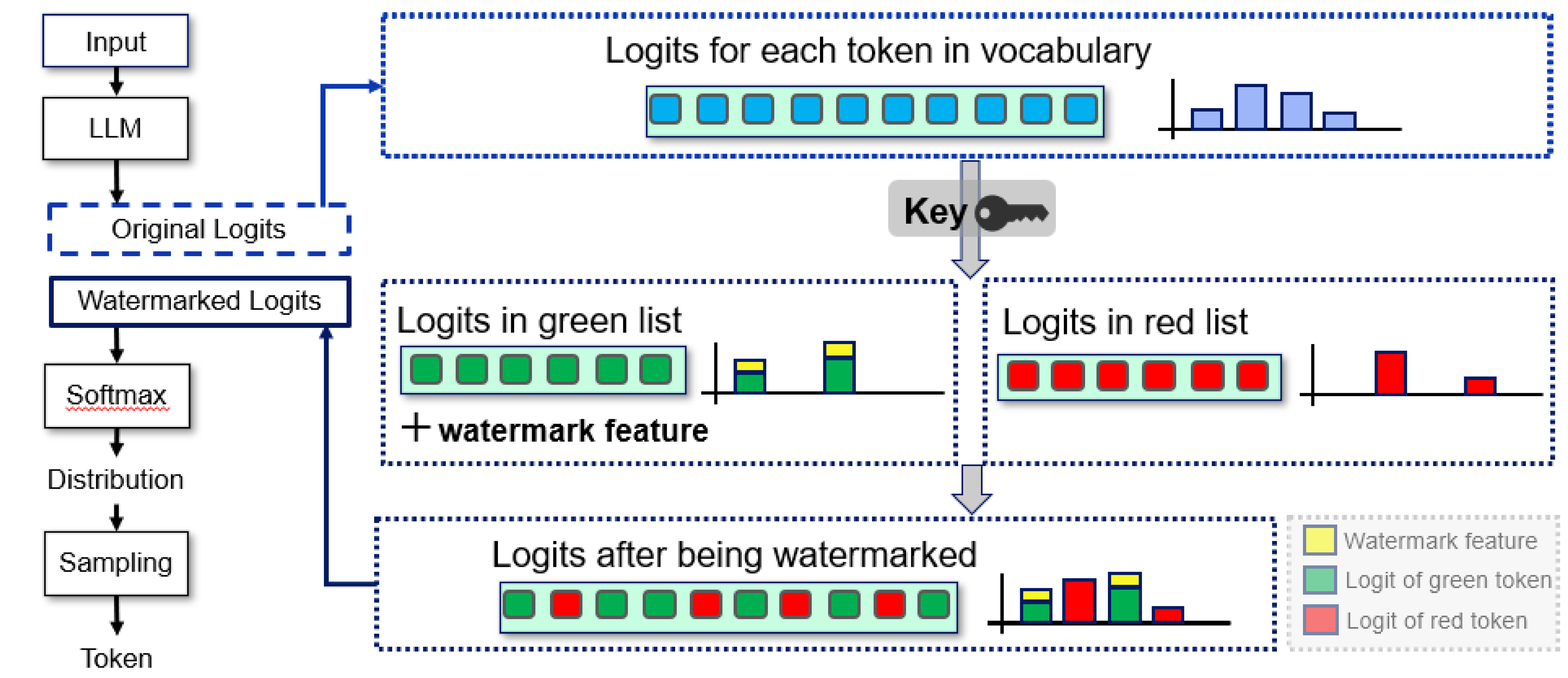
1. INTRODUCTION

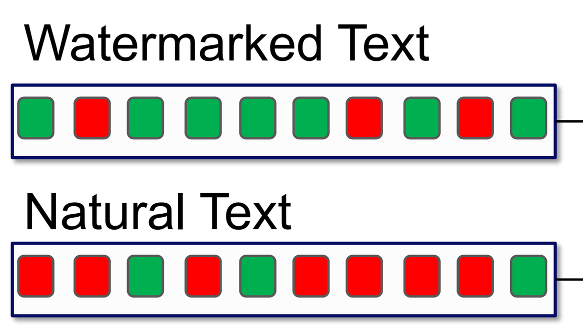
- LLM watermark is a novel technique to address copyright concerns, monitor AI-generated text, and prevent misuse.
- A watermark stealing attack aims to infer the details of an LLM watermarking scheme.
- Attackers can model watermark stealing as a constrained optimization problem to extract and remove the watermark without affecting text semantics.
- The proposed method can also be extended to handle multi-key watermark scenarios.

2. BACKGROUND: LLM WATERMARK

- **Adading Watermark into LLM**:
  Randomly divide the vocabulary into a green list and a red list, then raise the sampled probability by adding watermark features into the green list.



- **Watermark Detection**:
  After watermarking, the number of green tokens in the watermarked text is greater than in the non-watermarked text:



- Count the number of green tokens $G(S)$
- $G(S_{wm}) > G(S_{natural})$

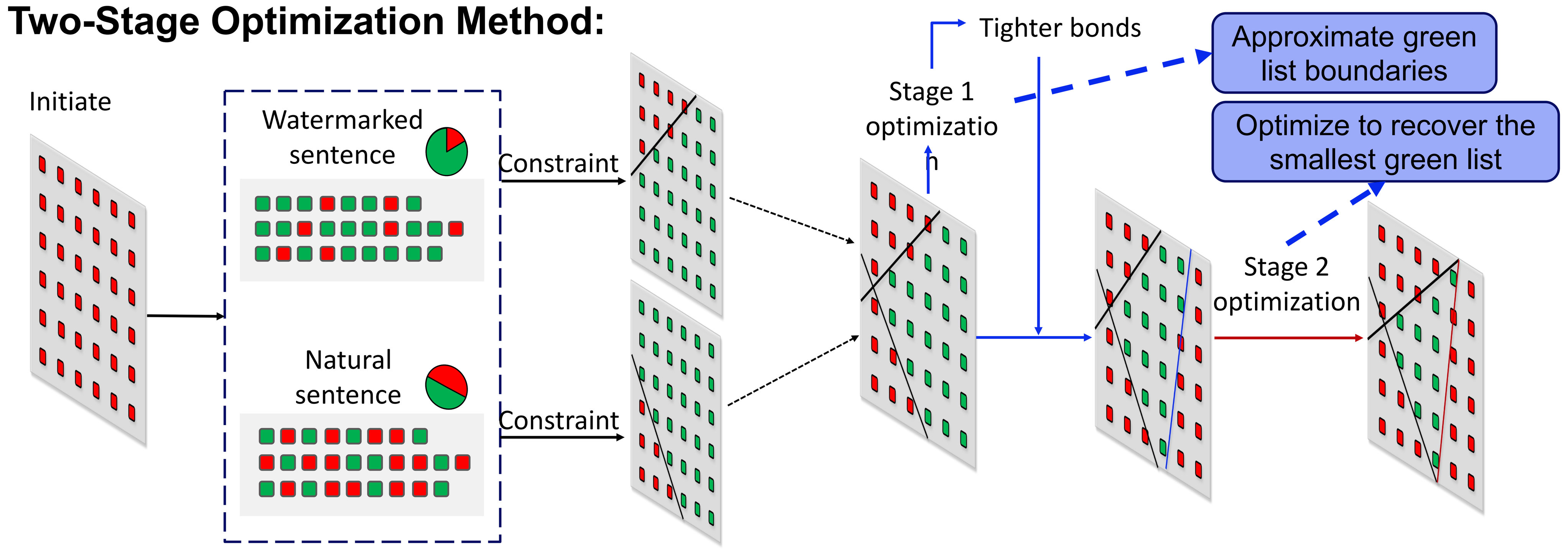
3. OUR METHODS: INSIGHTS

Watermark Stealing as an Optimization Problem:

- The association between tokens and the green list can be represented as integers;
- Constraints: watermark detection rules;
- Objective: finding a minimal available green list for the watermark text.

4. WATERMARK STEALING

- **Threat Models:**
  Attack Setting 1 (AS1) → attackers ***can*** access the watermark detector API.
  Attack Setting 2 (AS2) → attackers ***cannot*** access the watermark detector API.
- **Two-Stage Optimization Method:**

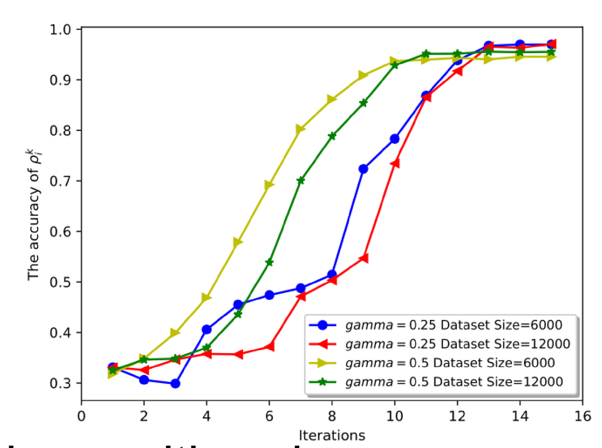


- **Multi-Key Stealing**: In this scenario, the attacker need to find suitable key for each sentence.

Random initialize $\{\hat{\rho}_i^k\}$
for until optimization converge **do**
  Optimize Stage-1 while fixing $\hat{\rho}_i^k$;
  Optimize Stage-2 while fixing $\hat{\rho}_i^k$;
  Resign $\hat{\rho}_i^k = \begin{cases} 1, \text{if } k = \operatorname{argmax}_k(G(S_i, k)) \\ 0, \text{else} \end{cases}$
end for

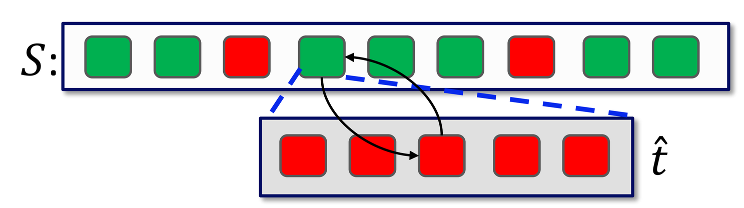
Which key is suitable for this sentence

The number of green token under key k



The accuracy of ρ while iterative optimization

- **Removing watermarks** in sentences by replacing green tokens with red ones.



Green token · Sentence S
Red token · Candidate Red token $\hat{t}$

5. RESULTS

- **Experimental Settings**: LLM: OPT-1.3B, LLaMA-2-7B; Dataset: C4; Solver for the Mixed Integer Programming: Gurobi.
- **Stealing performance** against LLaMA-2-7B under AS1 and AS2

Watermark setting	Dataset size	Ours (AS1) N_g	Ours (AS1) N_t	Ours (AS1) Precision(↑)	Freq. (AS1) N_g	Freq. (AS1) N_t	Freq. (AS1) Precision(↑)	Ours (AS2) N_g	Ours (AS2) N_t	Ours (AS2) Precision(↑)	Freq. (AS2) N_g	Freq. (AS2) N_t	Freq. (AS2) Precision(↑)
$\gamma = 0.25$, $\delta = 2$	4000	1064	885	83.18%	5154	2547	49.42%	3165	2003	63.29%	6032	2782	46.12%
	10000	1431	1224	85.53%	5519	2970	53.81%	2852	2069	72.55%	6613	3223	48.74%
	20000	1396	1256	89.97%	5494	3181	57.90%	2582	2056	79.63%	6727	3505	52.10%
	40000	2146	1912	89.10%	5425	3335	61.47%	2393	1990	83.16%	6680	3693	55.28%
$\gamma = 0.25$, $\delta = 4$	4000	732	678	92.62%	4350	2867	65.91%	3884	2813	72.43%	4392	2882	65.62%
	10000	780	731	93.72%	4704	3259	69.28%	4466	3347	74.94%	4736	3275	69.15%
	20000	867	803	92.62%	4895	3498	71.46%	4443	3481	78.35%	4937	3517	71.24%
	40000	933	861	92.28%	5020	3737	74.44%	4969	3923	78.95%	5062	3754	74.16%

- N_g: the number of tokens in the stolen green list
- N_t: the number of **true** green tokens in the stolen green list
- Precision= N_g/N_t
- Precision Average higher 18.23% / 9.52% in AS1 / 2

- **Removal performance** against LLaMA-2-7B under AS1 and AS2

Watermark setting	Dataset size	AS1 G_{avg}^b	AS1 $G_{avg}^a(\downarrow)$ Ours	AS1 $G_{avg}^a(\downarrow)$ Freq.	AS1 GRR(↓) Ours	AS1 GRR(↓) Freq.	AS2 G_{avg}^b	AS2 $G_{avg}^a(\downarrow)$ Ours	AS2 $G_{avg}^a(\downarrow)$ Freq.	AS2 GRR(↓) Ours	AS2 GRR(↓) Freq.
$\gamma = 0.25$, $\delta = 2$	4000	68.01	11.24	21.54	28.55%	52.56%	71.17	10.38	36.62	14.58%	51.46%
	10000	68.01	11.17	19.89	21.19%	50.84%	71.17	9.62	35.84	13.52%	50.35%
	20000	68.01	8.19	19.27	21.05%	50.37%	71.17	9.53	35.10	13.40%	49.32%
	40000	68.01	8.42	18.80	13.44%	50.41%	71.17	9.64	34.90	13.55%	49.04%
$\gamma = 0.25$, $\delta = 4$	4000	52.45	7.12	15.02	31.11%	47.81%	71.13	8.32	34.36	11.70%	48.30%
	10000	52.45	6.63	13.66	29.42%	47.49%	71.13	7.45	34.09	10.47%	47.92%
	20000	52.45	6.47	13.17	29.34%	48.35%	71.13	7.38	34.63	10.38%	48.68%
	40000	52.45	6.45	12.91	28.97%	48.81%	71.13	7.58	34.88	10.66%	49.04%

- G_{avg}^b: average number of green tokens ***before*** removal
- G_{avg}^a: average number of green tokens ***after*** removal
- GRR= G_{avg}^a/G_{avg}^b: the rate of remaining green tokens
- GRR Average lower 29.98% / 38.81% in AS1 / 2

https://github.com/plll4zzx/mip_watermark_stealing

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